Chapter 7
Transformations

Section 3
Rotations

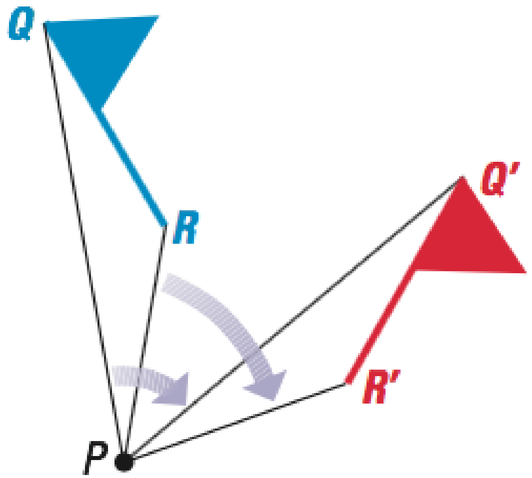
GOAL 1: Using Rotations

A **rotation** is a transformation in which a figure is turned about a fixed point. The fixed point is the **center of rotation.** Rays drawn from the center of rotation to a point and its image form an angle called the **angle of rotation.**

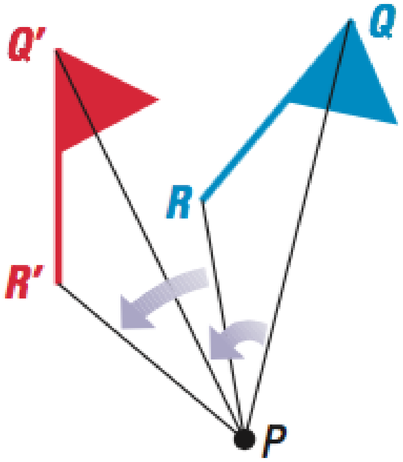
A rotation about a point P through x degrees ($x°$) is a transformation that maps every point Q in the plane to a point Q', so that the following properties are true:

1. If Q is not point P, then $QP = Q'P$ and $m\angle QPQ' = x°$.
2. If Q is point P, then $Q = Q'$.

Rotations can be clockwise or counterclockwise, as shown below.



Clockwise rotation of 60°



Counterclockwise rotation of 40°

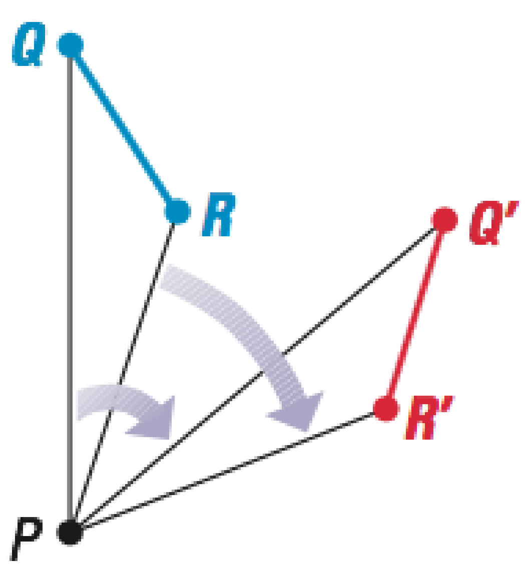
THEOREM

THEOREM 7.2 ***Rotation Theorem***

A rotation is an isometry.

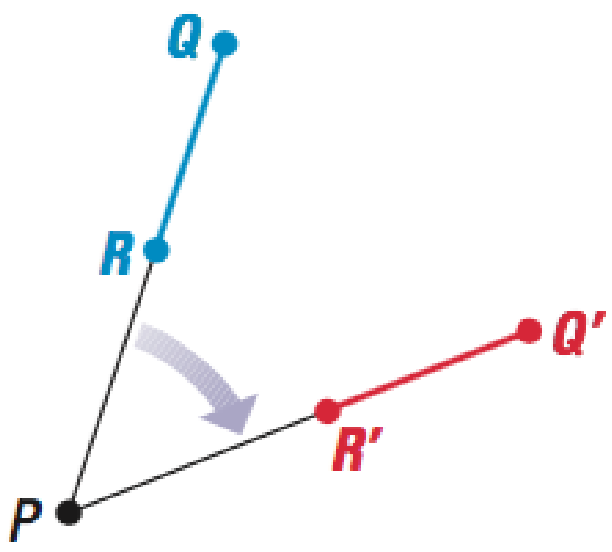
To prove the Rotation Theorem, you need to show that a rotation preserves the length of a segment. Consider a segment $\overline{QR}$ that is rotated about a point P to produce $\overline{Q'R'}$. The three cases are shown below. The first case is proved in Example 1.

CASE 1



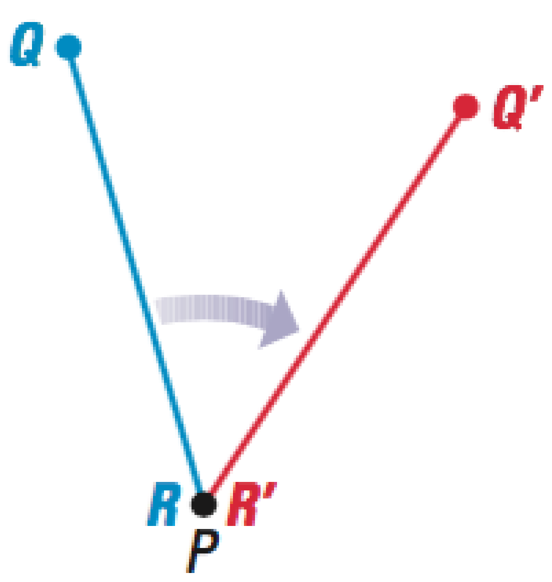
***R*, *Q*, and *P* are noncollinear.**

CASE 2



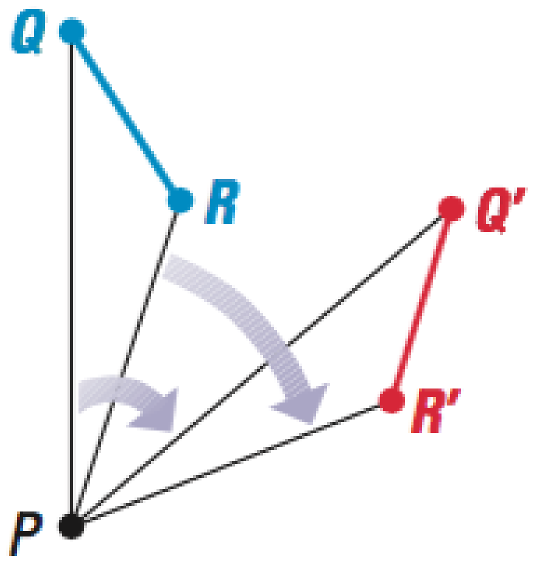
***R*, *Q*, and *P* are collinear.**

CASE 3



***P* and *R* are the same point.**

Example 1: Proof of Theorem 7.2



Write a paragraph proof for Case 1 of the Rotation Theorem.

GIVEN ▶ A rotation about P maps Q onto Q' and R onto R'.

PROVE ▶ $\overline{QR} \cong \overline{Q'R'}$

SOLUTION

Paragraph Proof By the definition of a rotation, $PQ = PQ'$ and $PR = PR'$. Also, by the definition of a rotation, $m\angle QPQ' = m\angle RPR'$.

You can use the Angle Addition Postulate and the subtraction property of equality to conclude that $m\angle QPR = m\angle Q'PR'$. This allows you to use the SAS Congruence Postulate to conclude that $\triangle QPR \cong \triangle Q'PR'$. Because corresponding parts of congruent triangles are congruent, $\overline{QR} \cong \overline{Q'R'}$.

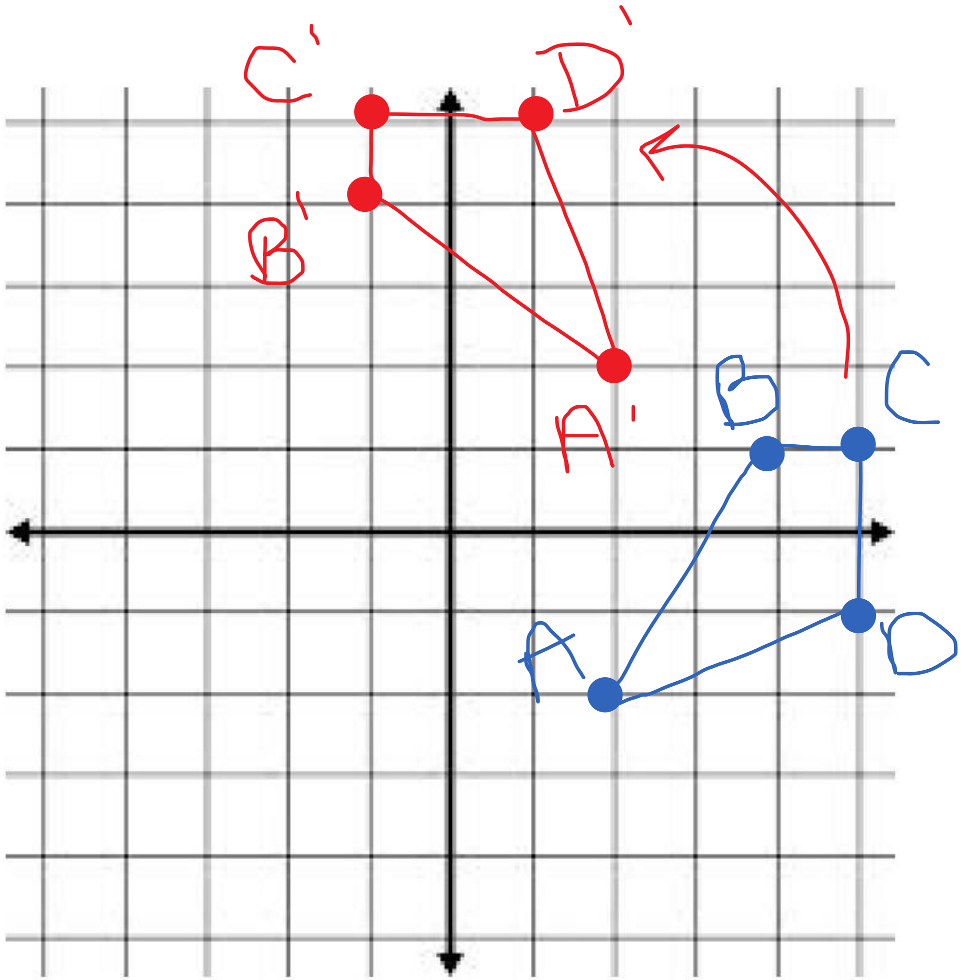
Example 2: Rotations in a Coordinate Plane

In a coordinate plane, sketch the quadrilateral whose vertices are $A(2, -2)$, $B(4, 1)$, $C(5, 1)$, and $D(5, -1)$. Then, rotate $ABCD$ 90° counterclockwise about the origin and name the coordinates of the new vertices. Describe any patterns you see in the coordinates.

Switch the x & y

Change the sign of

The "new" x



$A'(2,2)$

$B'(-1,4)$

$C'(-1,5)$

$D'(1,5)$

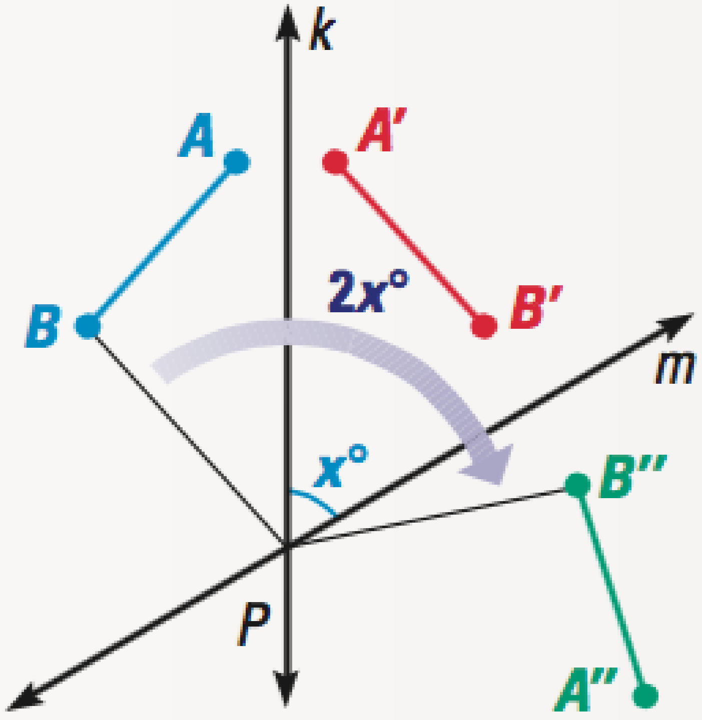
THEOREM

THEOREM 7.3

If lines k and m intersect at point P, then a reflection in k followed by a reflection in m is a rotation about point P.

The angle of rotation is $2x°$, where $x°$ is the measure of the acute or right angle formed by k and m.

$$m\angle BPB'' = 2x°$$



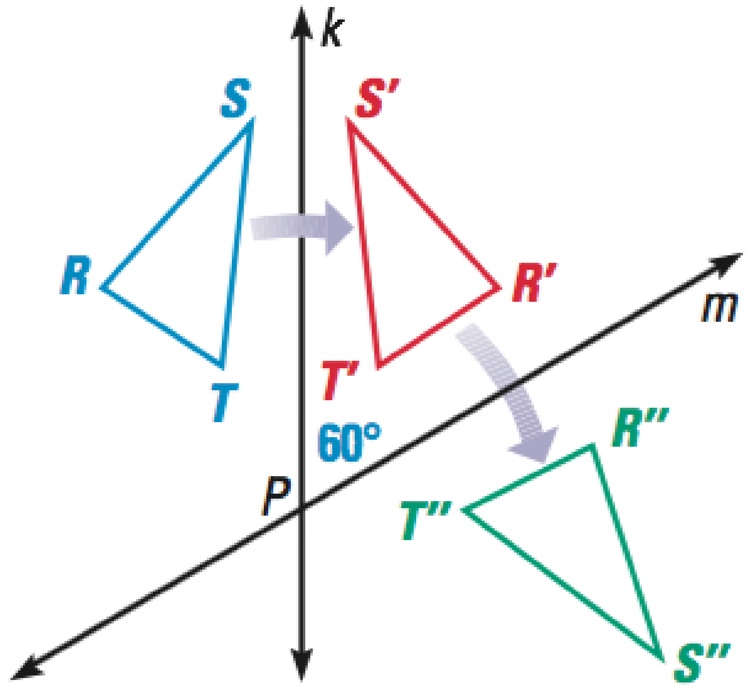
Example 3: Using Theorem 7.3

In the diagram, ΔRST is reflected in line k to produce ΔR’S’T’. This triangle is then reflected in line m to produce ΔR”S”T”. Describe the transformation that maps ΔRST to ΔR”S”T”.

$60 \times 2 = 120$

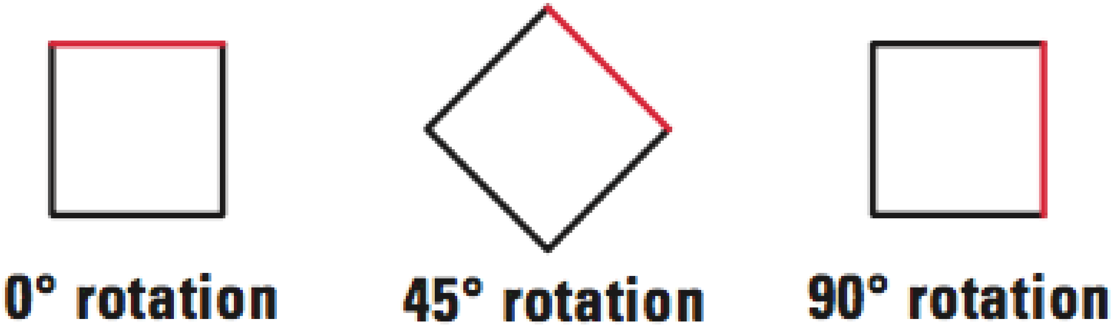
rotation

120° CW



GOAL 2: Rotations and Rotational Symmetry

A figure in the plane has **rotational symmetry** if the figure can be mapped onto itself by a rotation of 180° or less. For instance, a square has rotational symmetry because it maps onto itself by a rotation of 90°.

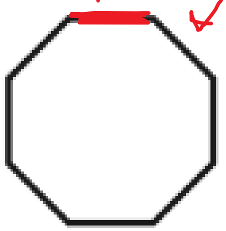


Example 4: Identifying Rotational Symmetry

180 or less

Which figures have rotational symmetry? For those that do, describe the rotations that map the figure onto itself.

a. Regular octagon

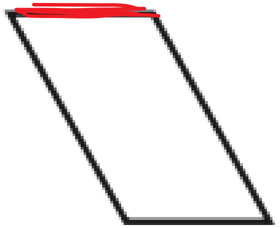


yes

$\frac{360}{8}$

45, 90, 135, 180

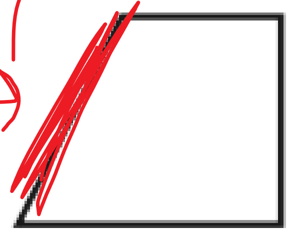
b. Parallelogram



yes

180

c. Trapezoid



no

Example 5: Using Rotational Symmetry

LOGO DESIGN A music store called Ozone is running a contest for a store logo. The winning logo will be displayed on signs throughout the store and in the store's advertisements. The only requirement is that the logo include the store's name. Two of the entries are shown below. What do you notice about them?

a.

180°

b.

$90^\circ, 180^\circ$

EXIT SLIP

Rotation Rules:

90* CCW (270* CW)

switch x & y, change sign of “new” x

180* CCW (180* CW)

change the sign of both x and y

270* CCW (90*CW)

switch x & y, change sign of “new” y